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# Formalising Istishab in Islamic Legal Theory: A Persistent Homology Model of Legal Continuity under Doubt

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A B S T R A C T

The presumption that an established legal status continues until a valid defeater appears (istishab) governs how Islamic jurists handle doubt, yet its structure over time has not been formalised. This qualitative-conceptual study maps the elements of istishab onto persistent homology, separating a normative admission rule for defeaters from a topological reading of continuity, and tests the model on four illustrative cases (ablution, ritual impurity, a missing person, and a contract with doubtful clauses). The initial status corresponded to a homology class that persisted until a valid defeater closed it, while doubt appeared as short-lived bars that left the class intact. Four barcode-reading rules give an auditable record of legal continuity without replacing ijtihad. Expert validation is the next step

## INTRODUCTION

Preserving a legal status under uncertainty is a central problem in usul al-fiqh, the principles of Islamic jurisprudence. One of the main tools for this problem is the presumption that a state or ruling already established continues to hold until evidence changes it, known as istishab (Al-Zuhayli, 1986). This presumption rests on the maxim al-yaqin la yazalu bi al-shakk (certainty is not removed by doubt), which al-Suyuti (died 911/1505) and Ibn Nujaym (died 970/1563) placed among the major legal maxims (Al-Suyuti, 1990; Ibn Nujaym, 1999). In usul al-fiqh, certainty and doubt are treated as graded states (yaqin, zann, shakk, and wahm), so a shift between states is not binary (Al-Zuhayli, 1986). Applications of istishab now extend to digital transactions and contracts (Hidayah et al., 2026). What has not yet been answered formally is how the continuity of a status is expressed when conflicting indications arrive in succession.

Studies that link Islamic law with mathematical formalisation are few and are dominated by fuzzy logic. Yüksek and Akın (2019) pointed to the resemblance between the grades of legal value of acts and graded logic, and Melynda et al. (2025) used Mamdani fuzzy inference to determine the inheritance rights of adopted children. Other studies have examined certainty and presumption in Shi'i usul (Mohaghegh Damad & Velaei, 2022) and the place of defeasible reasoning in usul al-fiqh (Komath, 2024).

The fuzzy approach expresses a graded quantity at a single moment through a membership degree (Zadeh, 1965). Applied to istishab, this approach would lower the degree of certainty whenever doubt appears, whereas the maxim holds that doubt does not displace established certainty. A threshold layer can avoid this, but the membership degree then no longer determines the decision, and time, which is the core of istishab, has no place in the formal apparatus. A tool that takes continuity and persistence as its main object is persistent homology, a method of algebraic topology that has developed within topological data analysis (Chazal & Michel, 2021, pp. 1–2; Herick et al., 2024, pp. 2327–2328; Otter et al., 2017, pp. 1–2). A search of the accessible literature found no study that uses algebraic topology to model istishab or any other usul al-fiqh reasoning.

This article aims to formulate a persistent-homology model of istishab and to show its structural correspondence with usul al-fiqh reasoning, using the integrative epistemology of Soleh (2023). Two questions guide the study: (1) how the elements of istishab (initial status, course of the case, doubt, and defeater) map onto the structure of persistent homology; and (2) how the model treats doubt and differences between schools in cases of worship, missing persons, and contemporary transactions. The novelty lies in applying persistent homology to istishab and in separating a normative layer, which decides what counts as a valid defeater, from a topological layer, which reads the continuity of the status. In this framework mathematics serves as a formal and analytical language, an interface between legal reasoning and computation. The model neither corrects nor replaces the authority of ijtihad, and it proposes no new legal ruling. Legal status remains binary, with topology merely reading the structure of its

continuity. In keeping with the integration–interconnection paradigm of the State Islamic University (UIN) Sunan Kalijaga Yogyakarta (Abdullah, 2006; Haidar & Dzulfahmi, 2024), the model is offered as a connection between an Islamic legal discipline and a mathematical one.

## LITERATURE REVIEW

### Earlier studies of istishab

Earlier studies of istishab have been doctrinal and applied. Haq (2017) showed that differences between schools lie in whether istishab preserves an existing ruling or creates a new one. In applied work, Putra (2021) discussed debt disputes and multi-contract transactions, Victoria et al. (2025) aligned the maxim al-yaqin with the principle of legal certainty, Sulfinadia et al. (2026) examined how Indonesian Religious Courts apply istishab in cases of missing heirs (mafqud), and Zangiabadi and Haji Dehabadi (2020) examined the order of istishab relative to the maxim of dar' (averting penalties through doubt) in criminal matters. These studies explain the content and application of the maxim, but they do not formulate the structure of continuity behind it, that is, how a status persists over time while doubt appears and disappears.

### The role of istishab in usul al-fiqh

In its classical formulation, istishab is expressed as al-asl baqa' ma kana 'ala ma kana: a state established in the past is presumed to remain as it was until a defeater appears (Al-Zuhayli, 1986). Al-Ghazali, al-Amidi, and al-Zarkashi discussed istishab and the disagreement over its probative force (Al-Amidi, 1982; Al-Ghazali, 1993; Al-Zarkashi, 2000). Scholars distinguish the original presumption of permissibility, the presumption of freedom from liability, and the presumption that a ruling or attribute established by evidence continues (Al-Zuhayli, 1986); the last type is closest to this article, because it concerns a status that has been proven and is now confronted with doubt. The standing of istishab as a proof is disputed: some schools accept it fully, some reject it, and some accept it only to preserve an existing ruling, not to establish a new one. The Hanafi school is generally associated with the last view, istishab as a proof for repelling (daf') and not for establishing (ithbat), while the majority gives istishab a wider function (Al-Zuhayli, 1986; Haq, 2017; Sulfinadia et al., 2026, pp. 134–135). Those who accept istishab, fully or partly, share the same logic of continuity: the starting point is an established ruling, not a conjecture, and change is valid only when caused by something that can be demonstrated.

### Defeasible reasoning and persistent homology

This structure of continuity is known in logic and philosophy as defeasible reasoning. Reiter (1980) called it default reasoning, that is, drawing conclusions as long as no contrary information is present, and Pollock (1987) distinguished defeaters that rebut a conclusion from those that undercut its basis. Popper (1959) stressed that a theory is held provisionally until contrary evidence rejects it, and Dierig (2021) re-examined this falsificationism by separating it from the conventionalist elements of Popper's main work. Komath (2024) showed that

usul al-fiqh already recognises such reasoning through analogy and dialectic, although his study does not address the continuity of status. Accordingly, istishab can be read as defeasible reasoning whose defeating rules are fixed normatively by the sharia. The next question is which formal tool can express that continuity.

Algebraic topology studies properties of shape that do not change under continuous deformation, and homology turns questions about shape into algebra. In a simplicial complex, a one-dimensional homology class represents a closed cycle that is not the boundary of any combination of the available 2-simplices (triangles), so that each class marks a hole (Chazal & Michel, 2021, pp. 9–10; Otter et al., 2017, p. 8). Persistent homology follows these classes along a growing sequence of complexes called a filtration and records when each class is born and when it dies (Chazal & Michel, 2021, pp. 13–15; Otter et al., 2017, pp. 11–12). The birth–death pairs are drawn as a barcode: long bars for features that persist and short bars for features that vanish quickly (Chazal & Michel, 2021, pp. 14–15). Topological data analysis reads short bars as noise and long bars as significant features. For readers of fiqh, four analogies help. A filtration is the order in which the facts and indications of a case come to light. A homology class is a status that stands because its supporting elements hold together. A barcode is the timeline of a case: a long bar is a status that stands, and short bars are doubts that come and go. A filling chain is the discovery of a valid defeater (mu'arid) that covers the whole supporting configuration and so ends the status.

## METHODS

This study is qualitative and conceptual, and it takes a comparative, interdisciplinary approach to usul al-fiqh and topological data analysis. It focuses on the inferential structure of istishab, that is, on how a legal status is maintained, doubted, and defeated, rather than on particular rulings. Its object is the structural similarity of reasoning across two traditions, so statistical testing was not required. The philosophical framework was the integrative epistemology of Soleh (2023), which requires a two-way encounter between religion and science on the principles of non-contradiction, equality, and mutual need. The study also followed the integrative-interconnective paradigm of UIN Sunan Kalijaga (Abdullah, 2006) in treating the two fields as distinct but connected. Within this framework, algebraic topology was treated as a method of reasoning parallel to the method of legal derivation (istinbat).

Primary fiqh sources were classical works on legal theory and legal maxims by al-Ghazali (died 505/1111) (Al-Ghazali, 1993), al-Amidi (died 631/1233) (Al-Amidi, 1982), al-Zarkashi (died 794/1392) (Al-Zarkashi, 2000), al-Suyuti (Al-Suyuti, 1990), and Ibn Nujaym (1999), together with the contemporary usul work of al-Zuhayli (Al-Zuhayli, 1986). Secondary sources included the studies of istishab cited in the Theoretical Review and the study of al-Ghazali's epistemology by Rofiq and Soleh (2025). Mathematical sources were recent reviews and computational studies of persistent homology (Chazal & Michel, 2021; Herick et al., 2024; Otter et al., 2017), including its stability results (Chazal & Michel, 2021, pp. 18–19). Two recent studies from other fields were read for how they handle screening, thresholds, and the reliability of a signal: a review of

macrohormone interference in endocrinology (Bronisławska et al., 2026) and a review of AI-text detection (Sharimbayev & Kadyrov, 2026). A draft textbook on persistent homology was read for biparameter and equivariant extensions (Frosini et al., 2026). Philosophical sources on reasoning were Popper (1959), Reiter (1980), Pollock (1987), and Dierig's recent re-examination of falsificationism (Dierig, 2021). Articles in Persian and German were consulted through their English titles and abstracts.

Sharia issues were analysed in three doctrinal steps before formalisation. First, the definition of istishab, its types, and its status as a proof were mapped from classical usul works and contemporary studies. Second, the grades of certainty and doubt and the kinds of evidence that the sharia accepts as defeaters were examined with examples from worship and transactions. Third, the results were checked against the opinions of the legal schools, and differences were treated as variation in the admission rule and in the mapping of istishab functions, not as errors of the model. The topological model was not used to issue rulings or to judge the correctness of any school.

The formal analysis was interpretive and conceptual, and it had three stages. In the first stage, concepts were mapped: initial status, course of the case, doubt, and defeater were translated into topological objects, namely filtered simplicial complexes, homology classes, and persistence intervals. In the second stage, rules were formulated: continuity of status was expressed as the survival of a homology class, and acceptance of a defeater as a separate normative rule. In the third stage, the model was tested on four illustrative cases: doubt after ablution, proven ritual impurity, the status of a missing person, and a contemporary contract with doubtful clauses. The barcode of each case was computed with the Geometry Understanding in Higher Dimensions (GUDHI) library, version 3.13 (Chazal & Michel, 2021, pp. 24–25; Otter et al., 2017, p. 25) using coefficients modulo 2, and the sensitivity of the model was tested against shifts in filtration values and variations in the admission rule (see the Results and Discussion sections). The case complexes were constructed by the authors from reconstructed fiqh scenarios, so the tests are illustrative and internal. The scripts used to construct and test the complexes (Python 3.12, GUDHI 3.13.0, random seed 2026) are provided as supplementary material; running python run_all.py reproduces Tables 3 and 4 and the tests reported in the Results. Validation by usul al-fiqh experts is planned as a later stage.

## RESULTS

### Structural Correspondence Between Istishab and Persistent Homology

Persistent homology corresponds to istishab in two respects. First, an established legal status is invariant: it persists while the surrounding indications fluctuate, and it disappears only through an event of a different kind, namely a valid defeater. Second, topology provides a language for distinguishing changes that merely shift a shape (deformation) from changes that remove its essential property (filling). The correspondence is structural. This article does not claim that classical scholars discovered topology or that istishab is a topological space;

it proposes that the structure of continuity in istishab can be modelled with this tool. Figure 1 summarises the correspondence used.

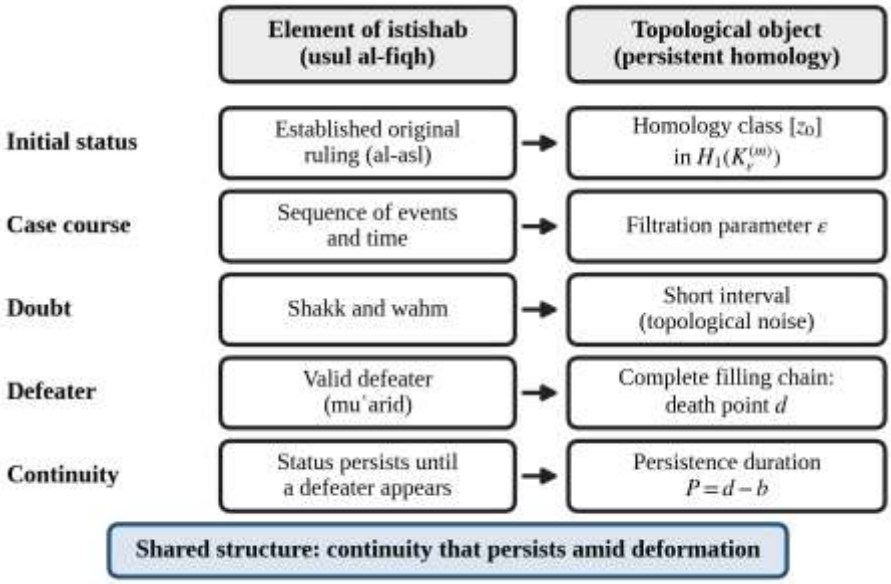



Figure 1. Structural Correspondence Between the Elements of Istishab and the Objects of Persistent Homology

Model formulation: case complex, supporting cycle, and admission rule

The model has two deliberately separated layers. The normative layer determines what may function as a defeater; it is the domain of scriptural evidence and ijtihad. The topological layer reads the continuity of the status from the structure of a case whose evidence has already been accepted; it is the domain of computation. Suppose a fiqh case is represented by a finite set of case elements: the facts that form the initial status (cause, condition, agent, object) and the indications that appear later (testimony, feelings, documents, reports). These elements are the vertices of a simplicial complex K. An edge (1-simplex) states that two elements are known to be related, and a triangle (2-simplex) states that three elements have been examined together and found to close on one another. Each simplex σ receives a filtration value f(σ), the point in time or order of events at which σ enters the case, with $f(\tau) \leq f(\sigma)$ whenever τ is a face of σ. The state of the case at filtration level ε is $K\varepsilon = \{\sigma \in K : f(\sigma) \leq \varepsilon\}$, so $K\varepsilon \subseteq K\varepsilon'$ whenever $\varepsilon \leq \varepsilon'$. Homology is computed with coefficients modulo 2, and the dimension of interest is one, written H1(Kε) (the first homology group of Kε), because legal status is represented by cycles.

Definition 1 (supporting cycle). The initial status S0 is represented by a supporting cycle z0, a closed loop of elements that support one another in establishing S0, for example the act of ablution, pure water, fulfilment of the pillars and conditions, and washing of the required body parts. The cycle is born at b, the largest filtration value among the simplices of z0, with $[z0] \neq 0$ in H1(K(m)b), where K(m) is the admitted complex of Definition 4. The hole inside the cycle indicates that no evidence has yet closed the whole configuration. The existence of this cycle corresponds to the usul condition that istishab is active

only when the initial status has been established with certainty (yaqin). The case in which the initial status is only a presumption (zann) is outside the scope of this article.

Definition 2 (persistence). The class [z0] is followed along the filtration K(m)ε of the admitted complex. It dies at d = inf{ε : [z0] = 0 in H1(K(m)ε)}, with d = ∞ if the class never dies within the observation window. The persistence duration is P = d − b. At an evaluation time ε′, the initial status is maintained if and only if b ≤ ε′ < d.

Definition 3 (noise, resolved doubt, and persistent doubt). Other birth–death intervals in the barcode that do not contain z0 are called non-anchor intervals. A doubt appears as a non-anchor cycle, and it is resolved when examination supplies 2-simplices that fill that cycle, for example a clarification showing that a sensation is consistent with a state of purity. Resolving a doubt differs from defeating a status: the filler of a doubt never has z0 as its boundary, whereas a defeater is a chain c with ∂c = z0. A non-anchor interval of duration at most τ is called topological noise. A non-anchor interval longer than τ that has died is read as resolved doubt, and one that is still open is called persistent doubt. The duration of an open interval is its age at the evaluation time, so an open interval that is not yet older than the threshold is pending and is not flagged. Either kind is recorded in the barcode but does not affect the decision. Intervals created by a rights-claim simplex, such as the reserved share in K3, are recorded as claim intervals and are not classified by duration. The threshold τ is a reading parameter, not a decision criterion: the decision depends only on whether [z0] is alive or dead. In all illustrative cases, τ = 2.

Remark 1 (calibration of the noise threshold by domain). The unit of the filtration parameter differs between domains (steps within an act of worship, years for a missing person, days in a transaction), so one value of τ cannot serve them all. For a domain D, let τD = max(2δD, Q1−α(RD)), where δD is the recording error of filtration values in D and RD is the time a harmless doubt takes to be resolved there, estimated from cases that experts have judged harmless. The first term follows from Proposition 2 below: intervals no longer than 2δD can appear or vanish under timing noise alone, so no reproducible label can depend on them. In the second term, α is the accepted false-flag rate, the share of harmless doubts that are still open after τD. The logic matches laboratory-specific cut-offs (Bronisławska et al., 2026, p. 3) and reporting at a fixed false-positive rate (Sharimbayev & Kadyrov, 2026, pp. 38148–38149). The threshold must be fixed before the cases are read, and it never changes the decision. Short-lived features are not noise in every domain (Onyango et al., 2025, p. 67), which is why the second term is estimated from the domain and not assumed.

Remark 2 (class and barcode). The decision is defined on the class [z0], not on a bar. Software pairs births and deaths by the elder rule, under which the younger of two classes that become homologous is the one that dies (Chazal & Michel, 2021, p. 15), even though neither class is zero. The barcode reading and the class reading therefore agree when the anchor is the oldest class, that is, when the initial certainty is established before any doubt arises. This is the order that the maxim itself presupposes (al-yaqin precedes al-shakk), and it holds in every

case of this article. When it does not hold, the decision should be read from the rank test of whether z0 lies in the image of the boundary map. A constructed counterexample shows the difference: a doubt cycle born before the anchor and later joined to it by a band of 2-simplices makes the barcode end the anchor's bar while the class remains non-zero. In 1,500 random complexes in which the anchor was the oldest class, the two readings never disagreed. In 736 complexes in which an older class existed, they disagreed in 240 (33%), always in the same direction: the barcode ended the anchor's bar while [z0] remained non-zero. Whenever the anchor is not the oldest class, the decision must therefore be read from the rank test and not from the bar.

Definition 4 (admission rule). For a school or interpretive framework m, $A(m)(\sigma) \in \{0, 1\}$ states whether a 2-simplex σ touching z0 may function as a filler, that is, a valid defeater. $A(m)(\sigma) = 1$ only if the source of σ is recognised by the sharia, for example direct sight or hearing, the testimony of two upright witnesses, or a judge's ruling, and its level of certainty satisfies the conditions of m. The complex analysed is K(m), namely K without the 2-simplices touching z0 that have $A(m) = 0$. This rule is set by scriptural evidence and ijtihad, not by computation. The mapping of istishab functions also belongs to this layer: for the function of repelling (daf') the model reads the class [z0] on the existing complex, and for the function of establishing (ithbat) the model allows a new rights-claim simplex resting on z0 only if the school in question accepts that function.

Two properties explain why topology suits istishab. Proposition 1 (complete filling). The class [z0] becomes zero in $K(m)\varepsilon$ if and only if there is a chain c of 2-simplices of $K(m)\varepsilon$ with $\partial c = z0 \pmod 2$, as follows from the definition of homology (Chazal & Michel, 2021, pp. 9–10). A defeater must therefore cover the whole supporting cycle. Indications that touch only some elements may shift the path of the cycle, but they do not remove its class. In an example where z0 has four elements, a new indication connected to three elements and forming two triangles only moves the path of the cycle, and [z0] remains non-zero; the same indication with four triangles closing all four edges makes [z0] zero.

Proposition 2 (stability against timing noise). Let f and g be two filtration functions on the same complex whose absolute difference does not exceed δ at any simplex, for example because the time of an indication was not recorded exactly. The bottleneck distance between the two persistence diagrams does not exceed δ (Chazal & Michel, 2021, p. 19; Otter et al., 2017, p. 22). The death point d of an anchor interval of duration greater than 2δ shifts by at most δ, and such an interval cannot vanish merely because of that shift, whereas intervals of duration less than 2δ may appear or disappear. This stability holds for shifts of filtration values, not for the addition of new filler simplices. Such an addition is a structural change governed by the admission rule, not by topology.

Corollary 1 (decision margin). Under the conditions of Proposition 2, the death time of the anchor class moves by at most δ, and so does its birth time. A decision taken at evaluation time ε′ is therefore robust to timing noise δ whenever the distance from ε′ to the death time exceeds δ. Proof: the sublevel sets satisfy $K_f(\varepsilon) \subseteq K_g(\varepsilon + \delta)$, and the same with f and g exchanged; a class that is zero in $K_f(d)$ maps to zero in $K_g(d + \delta)$, and symmetrically. This argument concerns the

anchor class itself, so it needs neither the matching of diagrams nor the elder rule. A scalar summary of the doubt in a case, the total persistence of the non-anchor intervals truncated at the evaluation time, is also stable: its change is at most 2δ times the larger number of intervals (Lee & Lee, 2024, p. 9).

Proposition 3 (certificates). At each evaluation time the decision can be accompanied by a certificate over F2. If [z0] = 0, a filling chain c of admitted 2-simplices with ∂c = z0 lists the simplices that defeat the status. If [z0] ≠ 0, there is a 1-cochain φ that vanishes on the boundary of every admitted 2-simplex and takes the value 1 on z0; it shows that no filling chain exists. Both exist by linear algebra, since a vector either lies in the image of a linear map or is separated from it by a functional that vanishes on the image, and both are checked by one pass of arithmetic modulo 2, without trusting the program that produced them. In 3,000 random cases (2,681 alive and 319 dead) every certificate verified.

Using these definitions, the model is applied to a case in five steps (Figure 2). The first step confirms that the initial status has been established and determines the supporting cycle z0 and its birth b. The second builds the case as a filtered complex from elements, relations, and time. The third sets the admission rule A(m) according to the school or ijtihad used. The fourth computes the H1 barcode by boundary-matrix reduction (Otter et al., 2017, pp. 18–20). The fifth reads the barcode: the initial status is maintained if [z0] is alive at the evaluation time, and abandoned from point d if [z0] is already dead. This sequence separates factual assessment (second step) from normative assessment (third step), so that disagreement can be traced to the relevant step.

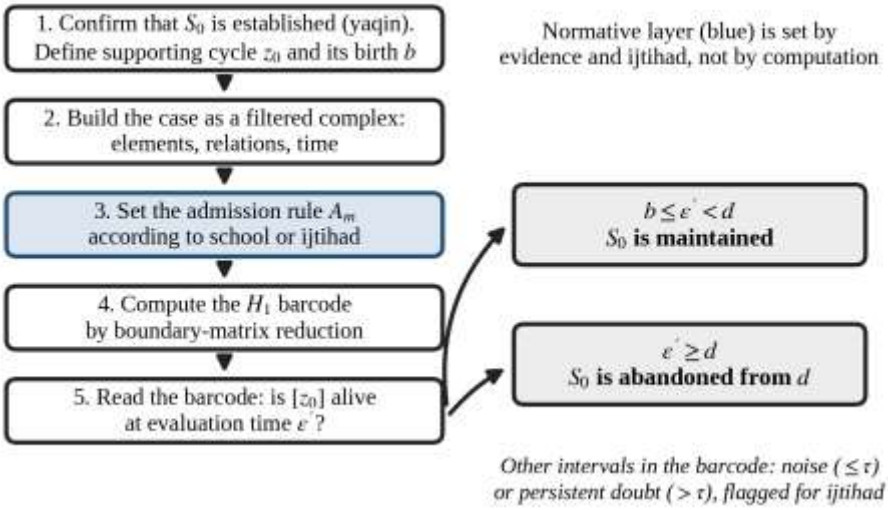



**Figure 2. Decision workflow for istishab based on persistent homology**

The barcode is read with four rules (Table 1). Rule R1 follows the maxim al-yaqin la yazalu bi al-shakk; R2 states that conjecture without proof, or an indication touching only some elements, does not suffice to defeat a status; R3 marks areas subject to ijtihad or caution; and R4 states that certainty collapses only through a valid defeater. R1 and R3 exclude each other, and R2 can accompany either. The differences between schools in accepting the functions of istishab, noted by Haq (2017), are the reason A(m) and the function mapping are

treated as parameters that may differ between schools, not as a single constant. The model does not unify differences of opinion; it shows where those differences operate.

Table 1. Barcode reading rules for istishab (z0 = supporting cycle of the initial status S0; τ = noise duration threshold; A(m) = admission rule of school m).

| Rule | Pattern on the barcode (IF) | Decision (THEN) | Basis |
|---|---|---|---|
| R1 | $[z_0]$ alive, and other non-anchor intervals are short ($\leq \tau$, still pending, or already resolved, for example indications of *shakk* or *wahm* that are quickly absorbed | $S_0$ maintained; short intervals recorded as noise | Maxim *al-yaqin la yazalu bi al-shakk* |
| R2 | $[z_0]$ alive although indications touch some elements of $z_0$, or filler simplices exist with $A_{(m)} = 0$ (source not recognised) | $S_0$ maintained | Conjecture without proof does not defeat a status |
| R3 | $[z_0]$ alive, and a non-anchor interval that has not died is older than $\tau$ | $S_0$ maintained; persistent doubt flagged for ijtihad or caution | Doubt does not displace the original ruling, but needs examination |
| R4 | A complete filling chain with $A_{(m)} = 1$ exists, and $[z_0]$ dies at $d$ | $S_0$ abandoned from $d$ | Certainty collapses only through a valid defeater |

**Model Specification and Transparency**

Table 2 lists the components of the model and the reasons for choosing them, so that the model can be examined and replicated

Table 2. Specification of The Persistent-Homology Model of Istishab.

| Component | Specification | Reason for the choice |
|---|---|---|
| Case elements | Vertices: facts forming the initial status and indications that appear later | Separates what is known from the assessment of it |
| Relations | Edge: two elements are related. Triangle: three elements examined together and closing on one another | Only a triangle can close a cycle |
| Filtration $f$ | Time at which a simplex enters the case (time or order of events) | Because istishab is continuity from the past, time is a natural parameter |
| Homology | $H_1$ with coefficients modulo 2 | Status is represented by cycles; modulo 2 avoids orientation issues |
| Anchor class | $[z_0]$, the supporting cycle of $S_0$ | Marks which interval represents the initial status |
| Admission rule | $A_{(m)}(\sigma) \in \{0, 1\}$ by school or interpretive framework | Keeps the authority of evidence and ijtihad over which defeaters are valid |
| Threshold $\tau$ | Duration limit for labelling noise; not a decision criterion | Only classifies non-anchor intervals |

| Component | Specification | Reason for the choice |
|---|---|---|
| Output | Binary decision ($S_0$ maintained or abandoned) with the barcode | Legal status remains binary; the barcode serves as an audit trail |
| Software | GUDHI for small cases; Ripser for large Vietoris–Rips complexes | Open tools allow replication |

The choice of homology dimension (H1) and of time as the filtration parameter reflects the thesis of this article, namely that istishab is a continuity of status that collapses only through a valid defeater. Legal status is represented by one-dimensional classes because such a class can be removed only by a filler that closes the whole cycle (Proposition 1), which parallels the maxim that certainty does not collapse under doubt that touches only part of the elements. These choices have a limitation: a standard filtration only allows the complex to grow, so a fact that is later withdrawn, such as retracted testimony, cannot be expressed directly. Such cases require an extension, for example zigzag persistent homology (Otter et al., 2017, pp. 22–24). Building the complex from textual data, for example a Vietoris–Rips complex over vector representations of fatwa texts, has not yet been done and is left for further research (Herick et al., 2024, p. 2328; Otter et al., 2017, p. 25).

**Case Studies: Worship, Missing Persons, and Contemporary Transactions**

Table 3 applies the model to four illustrative cases (K1–K4) and one variant of the function mapping (K3′), drawn from three domains: worship (K1 and K2), the status of a missing person or al-mafqud (K3), and contemporary transactions (K4). In all cases the initial status has been established with certainty, so istishab is active. The case complexes follow one construction rule: the supporting cycle z0 is a cycle of four elements whose simplices enter at ε = 0; a doubt is a vertex joined to two adjacent elements of z0, forming a small loop that one triangle resolves; and a defeater is a vertex joined to all four elements, whose four triangles fill z0. The value of ε in K1, K2, and K4 expresses the order of events, whereas ε in K3 is counted illustratively in years and must not be read as the time limit of any school. All barcodes were computed with GUDHI and are displayed in Figures 3 and 4.

**Table 3.** Application of the model to four illustrative cases ($H_1$ barcodes as birth–death pairs).

| Case | Main events ($\varepsilon$) | $H_1$ barcode | Decision |
|---|---|---|---|
| K1. Doubt after ablution | $z_0$ at $\varepsilon = 0$; sensation of moisture ($\varepsilon = 1$–4) and faint odour ($\varepsilon = 5$–8) each form a small loop that later closes | [0, ∞); [3, 4); [7, 8) | $S_0$ maintained (R1) |
| K2. Proven impurity | $z_0$ at $\varepsilon = 0$; direct hearing touches all four elements and closes them at $\varepsilon = 3$ | [0, 3) | $S_0$ abandoned from $\varepsilon = 3$ (R4) |

| Case | Main events ($\varepsilon$) | $H_1$ barcode | Decision |
|---|---|---|---|
| K3. Missing person, *daf'* and *ithbat* accepted | $z_0$ at $\varepsilon = 0$; each year without news ($\varepsilon = 1$–6); relative dies ($\varepsilon = 4$); judge's ruling ($\varepsilon = 8$) | [0, 8); [4, 8); six intervals $[k, k + 1)$ for $k = 1$–6 | Life maintained until $\varepsilon = 8$ (R1, R4); reserved share [4, 8) |
| K3′. Missing person, Hanafi function mapping | As K3, without the rights-claim simplex from the relative | [0, 8); six intervals $[k, k + 1)$ | Life maintained until $\varepsilon = 8$; no reserved share |
| K4. Contemporary contract with doubtful clauses | $z_0$ at $\varepsilon = 0$ (original permissibility); clause A ambiguous, then clarified ($\varepsilon = 1$–3); clause B unresolved ($\varepsilon \geq 2$) | [0, ∞); [1, 3); [2, ∞) | $S_0$ maintained (R3); clause B flagged |

K1: doubt after ablution. The supporting cycle z0 has four elements: the act of ablution, pure water, fulfilment of the pillars and conditions, and washing of the required body parts. The cycle is born at ε = 0. A sensation of moisture appears at ε = 1, connects to two elements of z0, forms a small loop at ε = 3, and closes at ε = 4, when examination supplies the triangles that fill the loop by showing that the sensation is consistent with a state of purity. A faint odour appears at ε = 5 with the same pattern, forms a second loop at ε = 7, and closes at ε = 8. The H1 barcode contains one long interval [0, ∞) and two intervals of duration one, [3, 4) and [7, 8). The long interval is the class [z0], so rule R1 applies: the state of purity is maintained, and both indications are recorded as noise. This agrees with the classical example in which a person who is certain of having performed ablution, and then doubts whether impurity has occurred, is still considered pure (Al-Suyuti, 1990; Al-Zuhayli, 1986).

K2: proven impurity. The supporting cycle is the same as in K1 but without the doubtful indications. At ε = 3, a report from direct hearing appears, touches all four elements of z0, and forms four triangles that close its four edges. Its source is recognised by the sharia (A(m) = 1) and the filling chain is complete, so [z0] becomes zero and the barcode contains one interval [0, 3). Rule R4 applies: the state of purity is abandoned from ε = 3. Comparing K1 and K2 shows a difference in kind, not in degree: in K1 both indications end as short bars at the bottom of the barcode, whereas in K2 a single event ends the main bar.

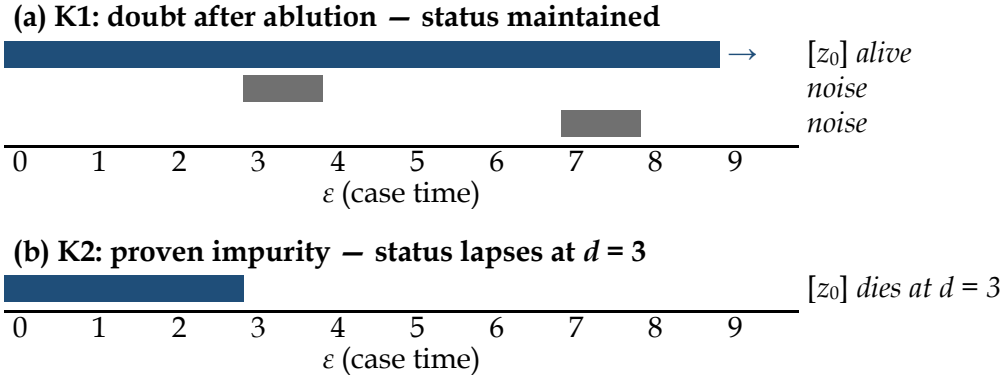



**Figure 3. H1 barcodes for K1 (doubt after ablution) and K2 (proven impurity), computed with GUDHI.**

K3: missing person. The initial status is life (hayat), that is, the person is still alive. The supporting cycle consists of the last known state of being alive, the attached legal relations (marriage and property rights), the absence of valid news of death, and the absence of a judicial determination. Each year without news (ε = 1 to 6) produces an indication of shakk that forms a loop lasting one year, since the absence of news is not proof of death. At ε = 4 a relative dies, creating a claim to the missing person's inheritance share, which is reserved. At ε = 8 a judge's ruling (qada' al-qadi) after investigation enters as one event touching all elements and is accepted as a defeater. Under a framework that accepts both the daf' and ithbat functions, the barcode contains [0, 8) for life, [4, 8) for the reserved share (a claim interval), which lasts as long as the life interval, and six short intervals that do not affect the decision. Treating the judge's ruling as the death point agrees with the way the cited sources describe the Shafi'i school, although the details of time limits and conditions differ between schools (Al-Zuhayli, 1986; Sulfinadia et al., 2026, pp. 125–127).

In the Hanafi school, a missing person is considered alive with respect to his own property but dead with respect to the rights of others (hayy fi haqqi nafsihi wa mayyit fi haqqi ghayrihi); this is a classic example of istishab used as a proof for repelling rather than for establishing (Al-Zuhayli, 1986; Haq, 2017; Sulfinadia et al., 2026, pp. 134–135). The model expresses this difference as a function mapping: under the daf' function, [z0] is read as it stands, so that the missing person's property is preserved while ε < 8, whereas under the ithbat function a rights-claim simplex from a relative may not rest on z0. The barcode of K3′ lacks the interval [4, 8), but the life interval [0, 8) is unchanged. The two frameworks differ in one interval, not in the whole structure.

K4: contemporary contract with doubtful clauses. In transactions, istishab works through the maxim that the original ruling of things is permissibility until evidence prohibits (Al-Suyuti, 1990). Suppose that a digital transaction-service contract has a supporting cycle of original permissibility consisting of a competent party, a clear benefit, consent, and the absence of prohibiting evidence. Clause A is initially worded ambiguously; at ε = 3 a clarification shows that it is consistent with the contract, so clause A leaves only the interval [1, 3). Clause B remains unresolved until the end of the observation window and forms a loop that never closes, the interval [2, ∞). The barcode has three intervals: [0, ∞) for the initial status, [1, 3) as noise, and [2, ∞) as persistent doubt. The status of permissibility is maintained, and clause B is flagged under R3 for examination through ijtihad or caution. The length of an interval does not automatically indicate the strength of evidence: the interval of clause B is long but is not a filler, so it does not defeat the status. The normative layer decides how clause B is handled (Hidayah et al., 2026; Putra, 2021).

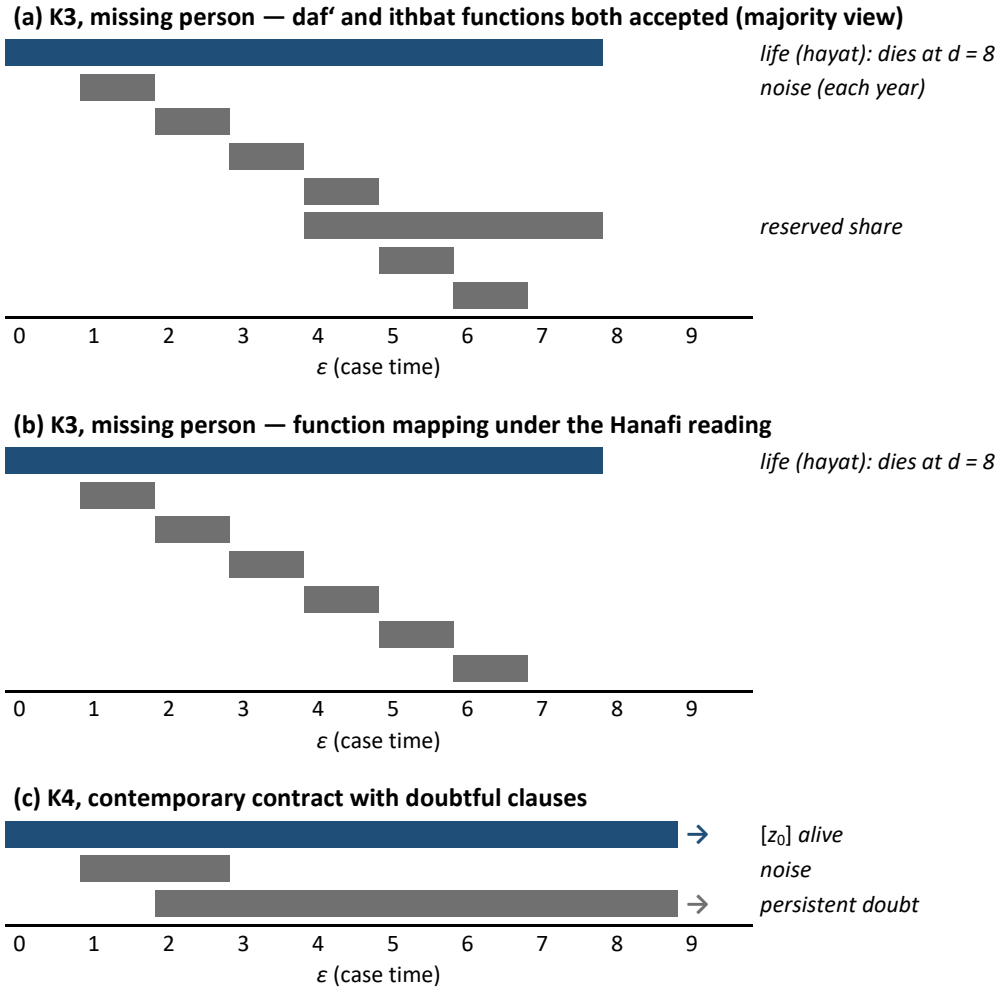



**Figure 4. H1 Barcodes For K3 (Missing Person, Two Function Mappings) And K4 (Contemporary Contract With Doubtful Clauses).**

## Sensitivity Tests and Variation of The Admission Rule

Sensitivity was tested in two ways. The first varied the admission rule and the complex structure to see when the decision changes. The second shifted filtration values randomly, while preserving the filtration condition, in 500 trials per case and per value of δ, to check Proposition 2. Table 4 summarises the results.

Table 4. Variations and Sensitivity Tests of the Model.

| Variation | Change | Result |
|---|---|---|
| V1 | K2 with $A_{(m)} = 0$: direct hearing replaced by conjecture, so the indication is not admitted and is removed with all its simplices | $[0, \infty)$; $S_0$ maintained (R2) |
| V2 | Partial K2: indication connected to three of four elements and forming two triangles | $[0, \infty)$; class survives with a shifted path, no new interval (R2) |
| V3 | K3 with the judge's ruling at $\varepsilon = 6$ | $d = 6$; reserved share becomes $[4, 6)$ |
| V4 | K3 without a judge's ruling | $[0, \infty)$ and the claim interval $[4, \infty)$; life maintained |

| Variation | Change | Result |
|---|---|---|
| V5 | K4 plus a definitive prohibition at $\varepsilon = 6$ | [0, 6); original permissibility lapses, clause B remains recorded as persistent doubt |
| V6 | Filtration shifts up to $\delta = 0.2$, 0.4, and 0.6 | Bottleneck distance did not exceed $\delta$ in any trial; death points of K2 and K3 shift by at most $\delta$; the anchor interval persists in all cases |

The decision depends only on whether a complete and accepted filling chain exists (V1, V2, V4, V5) and on the time at which that filler enters (V3). In V1 the indication is the same as in K2 and only its admission status differs, so the decision reverses from R4 to R2; the determinant lies in the normative layer. A non-admitted indication has to be removed with its vertex and all its simplices: removing only its 2-simplices leaves three open cycles that appear when the report enters, and the rules would wrongly flag them as persistent doubt. In V2, an indication touching only some elements does not remove the class, consistent with Proposition 1. V3 shows that a difference of ijtihad about when a judge's ruling takes effect only shifts the death point. In V6 the bottleneck distance never exceeded δ, consistent with Proposition 2. An extended run (300 trials for each of K2, K3, and K4 with a defeater, at three noise levels of 0.2, 0.4, and 0.6 time units) found no violation of the bound on the bottleneck distance or of the bound on the shift of the anchor's death time, and both bounds were reached to three decimals, so neither can be tightened.

Calibration of the threshold was tested on synthetic cases, not on real fatwa data. For each of three domains, 500 cases were generated, each with a Poisson number of harmless doubts (mean 2) and, with probability 0.5, one doubt that is never resolved. The resolution times were assumed to follow an exponential law with mean 1 step (worship), a normal law with mean 1 and standard deviation 0.25 years (missing person), and a lognormal law with mean 6 and standard deviation 5 days (transactions), with recording errors of 0.25, 0.5, and 2 time units (uniform within ± the stated value), observation windows of 20 steps, 10 years, and 60 days, and random seed 2026. Each doubt appeared at a time drawn uniformly from the window and was resolved after its sampled resolution time, or stayed open if that time fell beyond the window; the normal law was truncated below at 0.01, and the mean and standard deviation of the lognormal law refer to the distribution itself. The barcode of every case was computed with GUDHI, and a doubt still open at the end of the window was flagged when its age exceeded the threshold. With the fixed value 2, the false-flag rate for harmless open doubts was 18.6%, 1.1%, and 71.0% in the three domains, and the detection rate for never-resolved doubts was 89.2%, 73.2%, and 95.2%. With $\alpha = 0.05$ and thresholds of 3.35, 1.43, and 15.13 estimated from 300 calibration draws, the false-flag rates were 1.7%, 2.2%, and 1.9%, below α in all three domains, and the detection rates 83.8%, 80.0%, and 72.2%. Each domain contained about 59 to 107 harmless open doubts, so these rates carry a sampling error of up to about seven percentage points. The result shows what the procedure does, not what the true

resolution times of fiqh cases are; those must come from expert-judged cases. The supplementary scripts also reproduce the barcodes of K1–K4, K3′, and V1–V5 from the descriptions in this article.

These tests are internal: the authors built the case complexes, and no expert has yet assessed them. The proposed validation would involve five to seven usul al-fiqh experts from several schools, who would decide ten test cases independently. Their decisions would then be compared with the model output using a measure of inter-rater agreement such as Fleiss' kappa, giving priority to persistent-doubt cases such as K4, where the boundary between noise and a sign for ijtihad is hardest to draw.

## DISCUSSION

The study presents a topological model of istishab in Islamic jurisprudence (usul al-fiqh) to explain how established legal status remains valid amid uncertainty. It distinguishes ordinary doubt, which may be treated as noise, from persistent doubt requiring further examination and valid evidence that justifies changing a status. Unlike fuzzy logic, which is suitable for graded variables, the topological approach emphasizes continuity and persistence over time. The model complements existing doctrinal studies by formally representing legal decision-making while maintaining the authority of ijtihad and Islamic legal principles. It also supports interdisciplinary integration by connecting mathematical analysis with Islamic jurisprudence without allowing either discipline to dominate the other. Its main practical contribution is an auditable decision-support framework that records initial status, emerging evidence, persistent doubt, and the reasons for maintaining or changing a legal conclusion.

The model could support AI-assisted analysis of fatwa texts by combining language models for information extraction with deterministic topological verification. However, its application remains theoretical and requires expert validation, implementation, and testing against paraphrased cases and misleading evidence. Further research should address conflicts between legal maxims, withdrawn evidence, and the calibration of uncertainty thresholds. Researchers must also compare topological methods with simpler statistical approaches and report errors transparently. Ultimately, the model is intended to improve the transparency and consistency of Islamic legal decision-making, not replace qualified scholars or muftis, whose judgment and religious accountability remain essential.

## CONCLUSIONS AND RECOMMENDATIONS

Continuity of status in istishab can be modelled with persistent homology. An established initial status corresponds to a homology class, the course of the case to the filtration parameter, doubt to short-lived intervals, and a valid defeater to a filling chain that closes the supporting cycle so that the class dies at point d. The istishab decision is formulated as whether that class is alive or dead at the time of evaluation, so the maxim al-yaqin la yazalu bi al-shakk is reflected in the class's resistance to indications that touch only some elements and to timing noise. Scriptural evidence and ijtihad still determine the admission of

defeaters, and the differences between schools in the illustrative cases lie in the admission rule and in the mapping of istishab functions.

The contribution has three parts: (1) a structural correspondence between the elements of istishab and the objects of persistent homology, as an explicit language for legal continuity; (2) a separation of the normative and topological layers, plus four barcode-reading rules that can be discussed and tested among usul experts; and (3) an example of two-way integration under Soleh's framework that reduces neither usul al-fiqh nor mathematics and that operationalises the integration–interconnection paradigm. For Islamic legal studies, the model offers a way to talk about continuity and doubt explicitly without changing the content of the law, and it is an initial design for a transparent fatwa decision-support system.

**Recommendations for future research**

The recommendations focus on strengthening and validating the topological model of istishab and exploring its application in artificial intelligence for Islamic law. First, experts in usul al-fiqh should evaluate the model's assumptions, admission rules, and interpretation across different legal schools. Second, the formal framework should be extended to address withdrawn evidence, conflicts between legal maxims, and varying degrees of certainty. Third, the model should be implemented and tested using real fatwa texts. For AI development, researchers should establish a dedicated istishab benchmark, evaluate different reasoning systems, and compare topological methods with defeasible logic and language models. AI systems should also develop epistemic self-monitoring by recognizing levels of certainty and abstaining when evidence is insufficient. Furthermore, topological analysis could help detect inconsistencies in AI reasoning, particularly when numerous weak indications challenge an established legal status. These applications require rigorous testing, expert validation, and comparison with simpler methods. Overall, AI should function as a transparent decision-support instrument that assists qualified scholars and muftis while preserving human authority, religious accountability, and the central role of ijtihad in Islamic legal decision-making.

## FURTHER STUDY

This research is limited to conceptual analysis. The authors built the case complexes from reconstructed scenarios, and the choices of H1 and of a time filtration are modelling choices. A standard filtration cannot yet express withdrawn facts, and conflicts between istishab and other maxims such as dar' are not handled.

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